Comprobar para que valores del parámetro $\alpha$ la matriz:

$$A = \begin{pmatrix} \alpha & 0 & 1 \\ -2 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

es diagonalizable por semejanza en $\mathbb{R}$. Para $\alpha = 1$, si es posible, obtener la matriz diagonal $D$ y la matriz regular $P$ tal que $A = P\,D\,P^{-1}$.

$$p(\lambda) = |A - \lambda I| = \begin{vmatrix} \alpha-\lambda & 0 & 1 \\ -2 & -1-\lambda & 0 \\ 0 & 0 & 2-\lambda \end{vmatrix} = (\alpha-\lambda)(-1-\lambda)(2-\lambda) = 0$$

$$\Rightarrow \begin{array}{lcl} \alpha - \lambda = 0 & \Rightarrow & \lambda = \alpha \\ -1 - \lambda = 0 & \Rightarrow & \lambda = -1 \\ 2 - \lambda = 0 & \Rightarrow & \lambda = 2 \end{array}$$

- Si $\alpha \neq -1, 2$, todos los valores propios son distintos $\Rightarrow$ A es diagonalizable.

- Si $\underline{\alpha = -1}$

$\lambda_1 = -1$ (Doble) $\rightsquigarrow \alpha_1 = 2$

$\lambda_2 = 2 \rightsquigarrow \alpha_2 = 1 = d_2$

$$d_1 = 3 - rg(A - (-1)I) = 3 - rg(A + I) =$$

$$= 3 - rg \begin{pmatrix} 0 & 0 & 1 \\ -2 & 0 & 0 \\ 0 & 0 & 3 \end{pmatrix} = 3 - 2 = 1$$

$\alpha_1 = 2 \neq 1 = d_1 \rightsquigarrow$ A no es diagonalizable.

• Si $\alpha = 2$

$\lambda_1 = -1 \qquad \alpha_1 = 1 = d_1$

$\lambda_2 = 2 \qquad \alpha_2 = 2 \qquad d_2\ ?$

$$d_2 = 3 - rg(A - 2I) = 3 - rg\begin{pmatrix} 0 & 0 & 1 \\ -2 & -3 & 0 \\ \cancel{0} & \cancel{0} & \cancel{0} \end{pmatrix}$$

$$= 3 - 2 = 1$$

$\alpha_2 = 2 \neq 1 = d_2 \rightsquigarrow A$ ~~no~~ es diagonalizable

En resumen, $A$ es diagonalizable sii $\alpha \neq -1, 2$.

$\alpha = 1$

$$A = \begin{pmatrix} 1 & 0 & 1 \\ -2 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix} \qquad \left.\begin{array}{l} \lambda = 1 \\ \lambda = -1 \\ \lambda = 2 \end{array}\right\} \text{distintos}$$

$$D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

$$V_1 = Ker(f - 1.I) = \left\{ (x,y,z) \,/\, (A - 1.I)\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \right\}$$

$$\begin{pmatrix} 0 & 0 & 1 \\ -2 & -2 & 0 \\ \cancel{0} & \cancel{0} & \cancel{1} \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Longrightarrow \left.\begin{array}{l} z = 0 \\ -2x - 2y = 0 \end{array}\right\} \begin{array}{l} \to z = 0 \\ \to x = -y \end{array}$$

$$B_{V_1} = \{(-1, 1, 0)\} \Longleftarrow \left.\begin{array}{l} x = -\alpha \\ y = \alpha \\ z = 0 \end{array}\right\} \text{Ec. paramétricas}$$

$$V_{-1} = \{ (x, y, z) \,/\, (A - (-1)I) \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \}$$

$$\begin{pmatrix} 2 & 0 & 1 \\ -2 & 0 & 0 \\ 0 & 0 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \left. \begin{array}{r} \cancel{2x + z = 0} \\ -2x \quad = 0 \\ 3z = 0 \end{array} \right\}$$

$$\Downarrow$$

$$x = 0 \qquad z = 0$$

$$y = \alpha$$

$$B_{V_{-1}} = \{ (0, 1, 0) \}$$

$$V_2 = \{ (x, y, z) \,/\, (A - 2I) \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \}$$

$$\begin{pmatrix} -1 & 0 & 1 \\ -2 & -3 & 0 \\ \cancel{0} & \cancel{0} & \cancel{0} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \left. \begin{array}{r} -x + z = 0 \\ -2x - 3y = 0 \end{array} \right\}$$

$$\Downarrow$$

$$x = z$$

$$3y = -2x \Rightarrow y = \frac{-2}{3} x$$

$$\left. \begin{array}{l} x = \alpha \\ y = \frac{-2}{3} \alpha \\ z = \alpha \end{array} \right\} \text{Ec. param.}$$

$$B_{V_2} = \{ (1, -\tfrac{2}{3}, 1) \}$$

$$P = \begin{pmatrix} -1 & 0 & 1 \\ 1 & 1 & -\frac{2}{3} \\ 0 & 0 & 1 \end{pmatrix}$$

$$\underline{D = P^{-1} A P}$$